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A THEORETICAL STUDY OF A LONG BIELECTRIC-COATED CYLINDRICAL ANTENNA



By

Chung-Yu Ting

January 1967



Technical Report No. 517

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Cruft Laboratory

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ABSTRACT

By applying the Wiener-Hopf technique, the reflection coefficient of the transmission current at the end of a dielectric-coated antenna can be expressed in a single integral form. This result, when used with the solution of an infinite dielectric-coated cylindrical antenna, yields the input admittance and the current distribution of a long dielectric-coated antenna. It is found that unlike the locus of the admittance of a bare cylindrical antenna which converges to a point as the antenna gets longer and longer, the locus of the input admittance of the dielectric-coated antenna, becomes a circle. Also, due to the reflection of the transmission current back and forth, a standing wave with surface wave length is formed along the antenna.

I INTRODUCTION

The problem of the finite relatively short dielectric-coated cylindrical antenna has already been solved theoretically by a numerical method [1]. In principle, the method can be applied to an antenna of arbitrary length. However, due to the restricted number of storage locations available in a computer, the antenna length is also limited. To overcome this difficulty a new method is developed for a long dielectric-coated cylindrical antenna. It has been found [2] that for an infinitely long dielectric-coated cylindrical antenna, the current can be separated into two parts, the transmission current associated with the Goubau surface wave and the radiation current. If the coating is not very thin, the transmission current is much larger than the radiation current except very close to the source. Consequently, when the antenna is long enough the radiation current, which dies away very fast, can be neglected at the ends of the antenna. In this way, if the reflection coefficient of the transmission current at the end can be found, and it is assumed that the radiation current on the outside and the attenuated waveguide-mode current on the inside of the tube (which are both generated by the reflection of the incident transmission current at the end) die away very fast, then the characteristics of a long dielectric-coated cylindrical antenna can be determined.

II THE REFLECTION COEFFICIENT OF THE TRANSMISSION CURRENT

Fig. 1 shows a semi-infinite, perfectly-conducting tube terminated at z = 0 and imbedded in an infinite concentric dielectric cylinder. The conducting tube has the radius a, the dielectric cylinder the radius b and the dielectric constant ϵ_1 . By means of a Wiener-Hopf method similar to that used by Levine and Schwinger [3], the reflection coefficient can be found easily.

Assume there is an incident transmission current e^{ik_Bz} traveling down from $z=-\infty$ toward $z=\infty$, where k_B is the Goubau surface-wave number for the infinite structure. After the reflection of the incident current at z=0, the scattered current at r=a can be expressed as

$$I(z) = \begin{cases} Re^{-ik_S z} + g(z) & z \leq 0 \\ -e^{ik_S z} & z \geq 0 \end{cases}$$
 (1)

where R is the reflection coefficient of the transmission current, g(z) is some unknown function, and $-e^{-ik_Bz}$ cancels out the incident current e^{ik_Bz} . Since there is no conducting tube on the side z>0, there is no conduction current there.

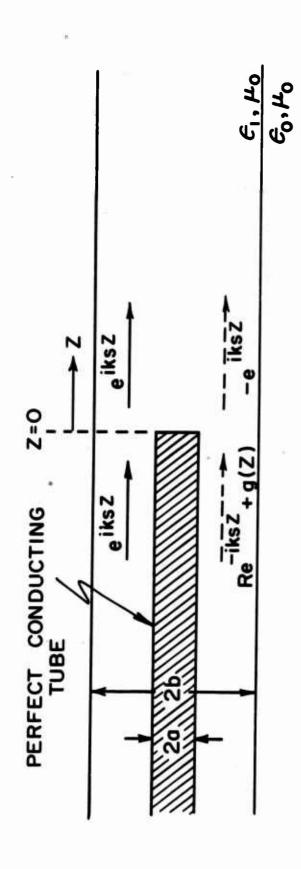
The boundary condition for a vanishing current at z = 0 is

$$R = -[1+g(0)]$$
 (2)

The Fourier transform of (1) is

$$\overline{I}(k) = \left[\frac{1}{i(k+k_s)}\right]_+ + \left[\frac{A}{i(k-k_s)} + \overline{G}_-(k)\right]_-$$
(3)

where the plus and minus signs indicate the plus and minus functions defined by



A SCHEMATIC DIAGRAM OF A SEMI-INFINITE PERFECTLY CONDUCTING TUBE IN AN INFINITE DIELECTRIC CYLINDER. FIG.

$$\overline{F}_{+}(k) = \int_{0}^{\infty} f(z)e^{ikz} dz$$

$$\overline{F}_{-}(k) = \int_{-\infty}^{0} f(z)e^{ikz} dz$$
(4)

 $\overline{F}_{+}(k)$ is analytic in the upper half k-plane and $\overline{F}_{-}(k)$ is analytic in the lower half k-plane. They have a common analytic region which shrinks to the real axis.

It has been found in [1] that the Fourier-transformed Green's function for the z-component of the electric field at r=a due to a ring delta current

source oriented in the z-direction i.e. $\frac{\hat{z}}{2\pi a} \delta(r-a) \delta(z)$ is

$$\frac{\{ \varepsilon_{r} \varphi H_{o}^{(1)} (\varphi b) [J_{o}(\xi a) Y_{1}(\xi b) -J_{1}(\xi b) Y_{o}(\xi a)]}{-i \xi^{2} J_{o}(\xi a) -\xi H_{1}^{(1)} (\varphi b) [J_{o}(\xi a) Y_{o}(\xi b) -J_{o}(\xi b) Y_{o}(\xi a)] \}}$$

$$\frac{K(k) = \frac{-i \xi^{2} J_{o}(\xi a) -\xi H_{1}^{(1)} (\varphi b) [J_{o}(\xi a) Y_{o}(\xi b) -J_{o}(\xi b) Y_{o}(\xi a)] \}}{4\omega \varepsilon_{1} [\xi J_{o}(\xi b) H_{1}^{(1)} (\varphi b) -\varepsilon_{r} \varphi J_{1} (\xi b) H_{o}^{(1)} (\varphi b)]}$$
(5)

where
$$\xi = \sqrt{k_1^2 + k^2}$$
 , $\phi = \sqrt{k_0^2 - k^2}$, $k_0^2 = \omega_1^2 \mu_0 \varepsilon_0$, $k_1^2 = \omega^2 \mu_0 \varepsilon_1$

J and H are Bessel and Hankel functions. The behavior of $\overline{K}(k)$ on the complex plane has also been discussed in [1]. It has two branch points at $k = \frac{+}{k_0}$ two simple zeros on the real axis at $k = \frac{+}{k_0}$ which

makes
$$\{\varepsilon r \phi H_o^{(1)}(\phi b) [J_o(\xi a) Y_1(\xi b) - J_1(\xi b) Y_o(\xi a)] - \xi H_l^{(1)}(\phi b) [J_o(\xi a) Y_o(\xi b) - J_o(\xi b) Y_o(\xi a)]\}_{k=-k_s} = 0$$

and an infinite number of zeros on the imaginary axis which make $J_o(\xi a) = 0$. There is no pole for $\sqrt{k_1^2 - k_0^2}$ b < 2, 405. The actual Green's function is obtained by the Fourier-inversing transform of (5). That is

$$K(z) = \frac{1}{2\pi} \int_{C} \overline{K}(k) e^{-ikz} dk$$
 (6)

The singularities of \overline{K} (k) and the path of integration c are shown in Fig. 2.

From the boundary condition that requires the tangential electric field to vanish on the surface of the perfect conductor, a Wiener-Hopf-type integral equation is formulated as follows:

$$\int_{-\infty}^{\infty} I(z') K(z-z') dz' = \begin{cases} 0 & z < 0 \\ E(z) & z > 0 \end{cases}$$
 (7)

Where E(z) is an unknown function of the tangential electric field for z > 0. The Fourier transformation of both sides of (7) combined with the assumption that E(z) is Fourier integrable, gives

$$\left[\frac{1}{i(k+k_s)}\right]_+ + \left[\frac{R}{i(k-k_s)} + \overline{G}_-(k)\right]_- = \frac{\overline{E}+(k)}{\overline{K}(k)}$$
(8)

Following the conventional Wiener-Hopf procedures, $1/\overline{K}$ (k) has to be split into a product of a plus and a minus function. Thus

$$\frac{1}{\overline{K}(k)} = \left[\frac{\overline{P}_{+(k)}}{k+k_{s}}\right]_{+} \left[\frac{\overline{Q}_{-(k)}}{k-k_{s}}\right]_{-}$$
(9)

 $1/\overline{K}(k)$ has two simple poles at $k = \frac{1}{2}k_s$. These have been separated out for simplicity.

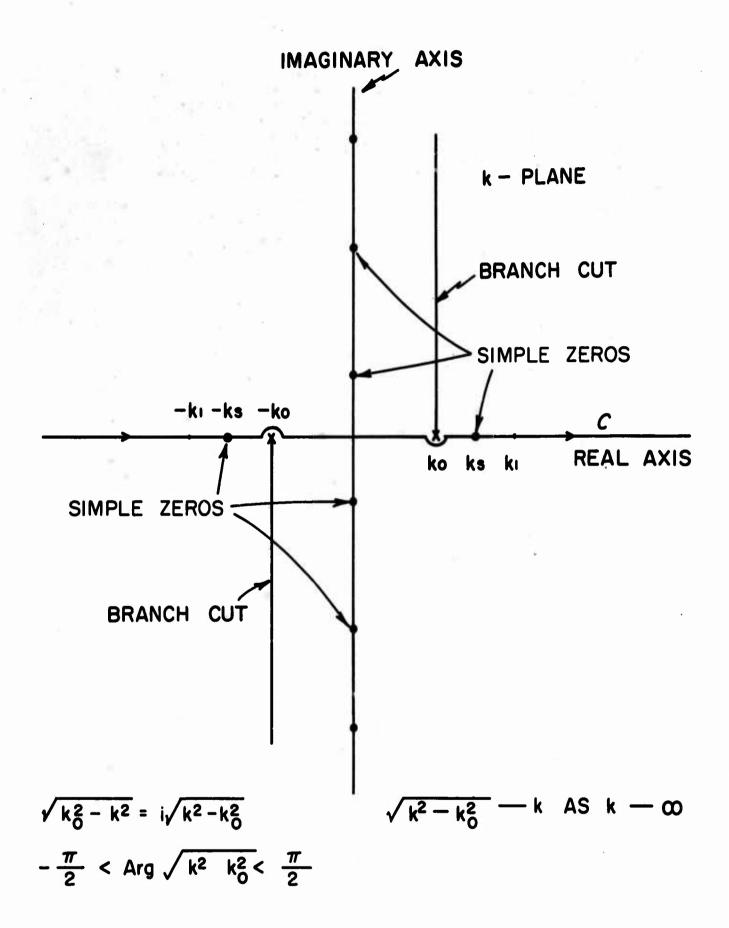


FIG. 2 SINGULARITIES OF K(k) AND INTEGRATION PATH C

The functions \overline{P} + (k) and \overline{Q} - (k) are

$$\overline{P}_{+} (k) = \exp \left[\frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{\ell_{n} \left(\frac{\lambda^{2} - k_{s}^{2}}{\overline{K}(\lambda)} \right)}{\lambda - k} d\lambda \right]$$

$$Im k \ge 0$$
(10)

$$\overline{Q}_{-}(k) = \exp \left[\frac{-1}{2\pi i} \int_{-\infty}^{\infty} \frac{\ell_{n} \left(\frac{\lambda^{2} - k_{s}^{2}}{R(\lambda)}\right)}{\lambda - k} d\lambda\right]$$

$$Im \ k \le 0$$
(11)

Let [9] be rewritten as follows:

$$\frac{\mathbf{k} - \mathbf{k}_{s}}{\mathbf{i}(\mathbf{k} + \mathbf{k}_{s})\overline{Q}_{-}(\mathbf{k})} + \left[\frac{\mathbf{R}}{\mathbf{i}\overline{Q}_{-}(\mathbf{k})} + \frac{(\mathbf{k} - \mathbf{k}_{s})\overline{G}_{-}(\mathbf{k})}{\overline{Q}_{-}(\mathbf{k})}\right]_{-}$$

$$= \left[\frac{\overline{E}_{+}(\mathbf{k})\overline{P}_{+}(\mathbf{k})}{\mathbf{k} + \mathbf{k}_{s}}\right] \tag{12}$$

Then, by splitting the first term of (12) into a sum of a plus and a minus function, (12) becomes

$$\left[\frac{k-k_{s}}{i(k+k_{s})\overline{Q}_{-}(k)} + \frac{2k_{s}}{i(k+k_{s})\overline{Q}_{-}(-k_{s})}\right]_{-} + \left[\frac{R}{i\overline{Q}_{-}(k)} + \frac{(k-k_{s})\overline{G}_{-}(k)}{\overline{Q}_{-}(k)}\right]_{-}$$

$$= \left[\frac{E_{+}(k)\overline{P}_{+}(k)}{k+k_{s}}\right]_{+} + \left[\frac{2k_{s}}{i(k+k_{s})\overline{Q}_{-}(-k_{s})}\right]_{+} (13)$$

The left side of (13) is a minus function, the right side is a plus function, therefore they must be equal to an entire function. From an investigation of

the asymptotic behavior of the functions on the right in (13), it is easy to prove that the entire function is zero. Since as $|k| \to \infty$, $\overline{K}(k) \sim k$, $\overline{K}(k)$ is an even-function of k so that $\overline{P}_+(k) = \overline{Q}_-(-k) \sim \sqrt{k}$ and $\overline{E}_+(k)$ is no worse than a constant, the right side of (13) tends to zero. It follows that

$$\frac{R}{i(k-k_{s})} + \overline{G}_{k}(k) = -\frac{1}{k-k_{s}} \left[\frac{k-k_{s}}{i(k+k_{s})} + \frac{2k_{s} \overline{Q}_{k}(k)}{i(k+k_{s}) \overline{Q}_{k}(k)} \right]$$
(14)

When the inverse Fourier transforms are taken on both sides of (14), the residue contributions due to the simple pole at k=k, should be equal on the two sides of the equation, which gives

$$R = -\frac{\overline{Q}_{-}(k_{g})}{\overline{Q}_{-}(-k_{g})}$$
 (15)

Eq. (15) gives a simple expression of the reflection coefficient of the transmission current. The next problem is to evaluate \overline{Q} (kg) and \overline{Q} (-kg). It can be shown that \overline{Q} (k) of (11) can be rearranged into the following form:

$$\overline{Q}_{-}(k) = \exp\left[\frac{-k}{\pi i} \int_{0}^{\infty} \frac{\frac{\lambda^{2} - k_{s}^{2}}{R(\lambda)}}{\lambda^{2} - k^{2}} d\lambda\right]$$

$$Im \ k < 0$$
(16)

As k approaches the real axis (16) can be separated into two parts, the principle value of the integration and the half residue picked up at $\lambda = k$. With $k = k_s$, and $-k_s$, respectively, in (16), a final form of (15) is

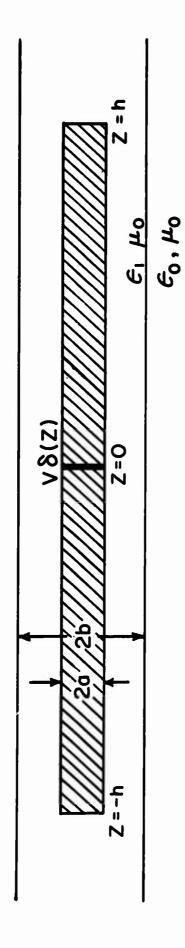
$$R = -\exp\left[\frac{-2k_s}{\pi i} P \int_0^\infty \frac{k_n(\frac{\lambda^2 - k_s^2}{K(\lambda)})}{\lambda^2 - k_s^2} d\lambda\right]$$
 (17)

where P indicates the principle value. (17) can be computed numerically by the computer, and in general R is a complex quantity.

III LONG DIELECTRIC-COATED CYLINDRICAL ANTENNA

The above result can be readily applied to a long dielectric-coated antenna. It has been shown [2] that when the dielectric coating is reasonably thick, the radiation current excited by a delta generator is much smaller than the surface-wave current, except very close to the generator. Also its rate of decay is faster than expontentially initially; it becomes 1/z² asymptotically. Therefore, when the antenna is long enough the radiation current can be neglected at the ends of the antenna. The small radiation current, the cut-off wave-guide-mode current generated by the incidence of the transmission current upon the ends of the antenna i. e. g(z)in (1) is also assumed to decay very fast. When this is true the radiation and transmission currents can be considered separately. Fig. (3) shows a long dipole imbedded in an infinitely long dielectric cylinder. The antenna, with length 2h, is driven by a delta generator at z = o. Let the reflection coefficient of the transmission current at the ends be R, then the current at any point z can be considered as the sum of the radiation current, the infinite series of the multiply reflected transmission currents and the unknown reflected current g(z). Mathematically it can be written in the form

$$I(z) = I_{r}(z) + G_{s} - \frac{e^{ik_{s}z} \left[1 + Re^{i2k_{s}(h-z)}\right] + e^{ik_{s}h} g(z-h)}{1 - Re^{i2k_{s}h}}$$
(18)



A SCHEMATIC DIAGRAM OF A LONG DIPOLE ANTENNA IN AN INFINITE DIELECTRIC CYLINDER n FIG

where $I_r(z)$ is the radiation current of an infinitely long dielectric-coated antenna, and G_s is its input transmission conductance. The current-standing-wave ratio is found to be

$$S = \frac{1+|R|}{1-|R|} \tag{19}$$

Let the input admittance be defined as the current at point z = 0 which, from (18), is given by

$$Y_{in} = G_r + iB_r + G_s \frac{1 + Re^{i2k_sh}}{1 - Re^{i2k_sh}}$$
 (20)

where G_r and B_r are the input radiation conductance and input radiation susceptance. If the real and imaginary parts are separated, the input conductance is

$$G_{in} = G_{r} + \frac{G_{s}(1-|k|^{2})}{1+|k|^{2}-(Re^{i2k_{s}h}+R^{*}e^{-i2k_{s}h})}$$
(21)

and the input susceptance is

$$B_{in} = B_r + \frac{-i G_s (Re^{i2k_sh} - R^*e^{-i2k_sh})}{1 + |R|^2 - (Re^{i2k_sh} + R^*e^{-i2k_sh})}$$
(22)

If the resonant and antiresonant lengths of the antenna are defined respectively at the maximum and minimum of the input conductance, these lengths are determined by $\frac{\partial G}{\partial h} = 0$.

This gives

$$h = \frac{-1}{2k_s} \left(\tan^{-1} \frac{b}{a} \right) \tag{23}$$

where a, and b are the real and imaginary parts of R. The corresponding values of G_{in} are

$$G_{in} (max) = G_{r} + S \cdot G_{s}$$
 (24)

$$G_{in}^{\cdot} (min) = G_{r}^{\cdot} + G_{s}/S$$
 (25)

Similarly the maximum and minimum of Bin occur where $\frac{\partial B_{in}}{\partial h} = 0$. This gives

$$h = \frac{1}{2k_s} \left(\cos^{-1} \frac{2|k|}{1+|k|^2} - \tan^{-1} \frac{b}{a} \right)$$
 (26)

It is also noted, that the locus of the input admittance on the complex admittance plane is a circle.

IV NUMERICAL RESULTS

In the numerical calculation the input radiation susceptance of an infinitely long dielectric coated antenna, B_r , due to a delta generator is infinite. One way to avoid this difficulty is to subtract the inside current from the outside current. Since the same logarithmic singularity occurs on both inside and outside surfaces near the driving point, these cancel when the two currents are subtracted, a finite value is obtained at z=0.

This is given by

$$\xi H_{1}^{(1)}(\phi b)[Y_{o}(\xi b)J_{1}(\xi a)-J_{o}(\xi b)Y_{1}(\xi a)] \qquad (27)$$

$$Gr+iBr=ia\omega\epsilon_{1}VP\int_{c}\left[\frac{-\varepsilon r\phi H_{o}^{(1)}(\phi b)[Y_{1}(\xi b)J_{1}(\xi a)-J_{1}(\xi b)Y_{1}(\xi a)]}{\xi\{\xi H_{1}^{(1)}(\phi b)[J_{o}(\xi a)Y_{o}(\xi b)-J_{o}(\xi b)Y_{o}(\xi a)]}+\frac{J_{1}(\xi a)}{\xi J_{o}(\xi a)}\right]dk$$

$$-\varepsilon r\phi H_{o}^{(1)}(\phi b)[J_{o}(\xi a)Y_{1}(\xi b)-J_{1}(\xi b)Y_{o}(\xi a)]\}$$

Although this finite value does not necessarily correspond exactly to the actual value for an infinite antenna with a certain gap, they have the same order of magnitude as has been checked experimentally [4].

The numerical values for the three cases, $\epsilon_r = 3.0$, b/a = 2, 4, 8, have been calculated, the results are listed in the following table

b/a	2	4	8
k _s /k _o	1.056881	1.143743	1. 279899
Current Reflection Coefficient R	-0.515-i 0.474	-0.489-i 0.577	-0. 451-i 0. 627
Current Standing Wave Ratio S	5.76	7. 20	7.81
Resonant Length $/\lambda_0$	0.180+n x 0.473	0.158+nx 0.437	0.134+n×0.391
$G_r + iB_r$	0. 763+i 3. 26	0.652+i 3.19	0. 656 +i 3. 21
$G_{f g}$	3. 29	2. 94	3. 99
G _{in(max)}	13.75	21.81	32, 20
G _{in(min)}	1.16	1. 16	1, 17

where n is a large integer.

The loci of the input admittances expressed by (21) and (22) on the complex plane for the above three cases, as the antenna length h is varied, are shown graphically in Fig. 4. Superimposed on each circle is the input admittance curve for a dielectric-coated antenna as calculated from [1] by a numerical method and corrected (imaginary part) according to Lamensdorf's experiment [1] [4]. It is interesting to note that as the length of the antenna increases, it approaches the circle calculated from (21) and (22). Just as for a transmission line with a fixed load, the locus of the input admittance is a circle, as the length of the transmission line is increased. Note that this differs from the bare long dipole antenna [5], for which, because of the decaying current, the locus ultimately converges to one point as the antenna is made longer and longer. Evidently, the long dielectric-coated antenna behaves primarily like a transmission line.

Typical current distributions in both magnitude and phase angle as obtained from (18) for the above three cases and with $h = 3\lambda_0$ are shown in Fig. 5. The contribution by the radiation current of an infinite dielectric-coated antenna is obtained from [2]. Since part of the reflected current g(z) is unknown, the dotted line at the ends are drawn arbitrarily. Nevertheless since g(z) should decay very rapidly its influence is confined to small regions near the ends. Also note that the amplitude of the current when b/a = 2 is much larger than when b/a = 4, 8 because the structure is near the resonant length. With the thicker dielectric cylinders the structures are close to anti-resonance.

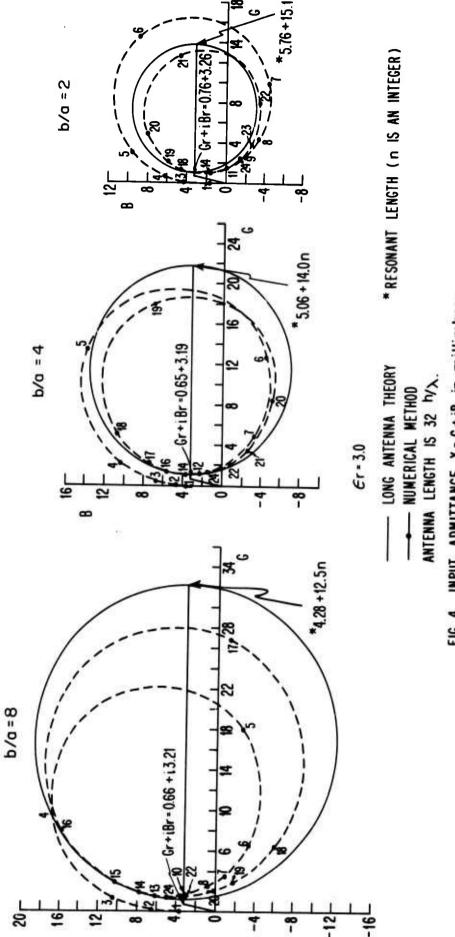
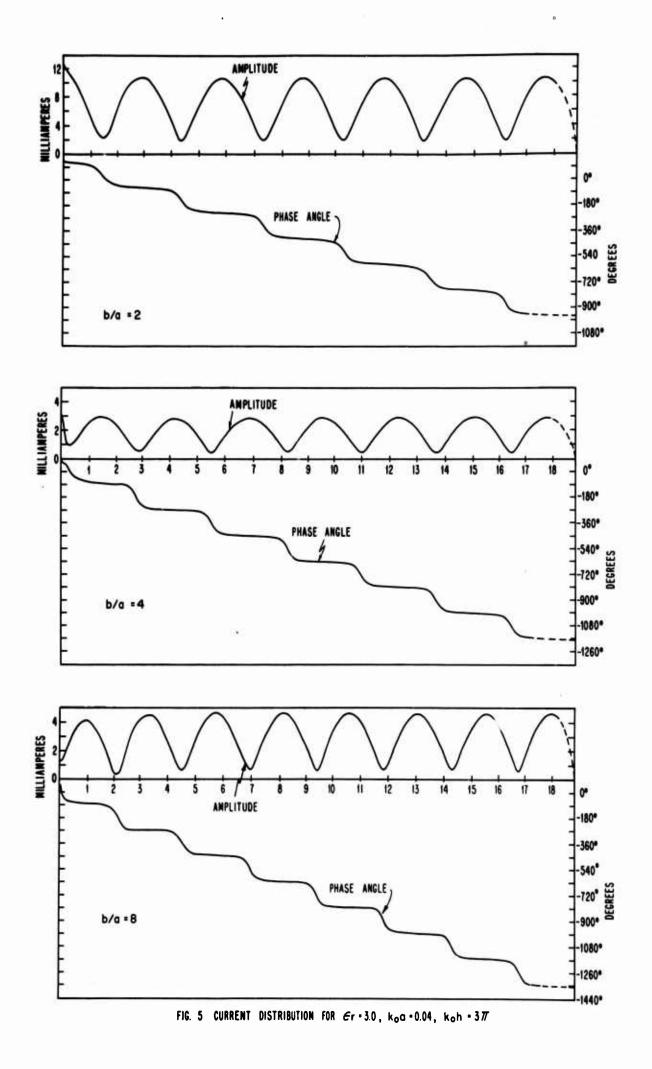


FIG. 4 INPUT ADMITTANCE Y . C + iB in millimhos

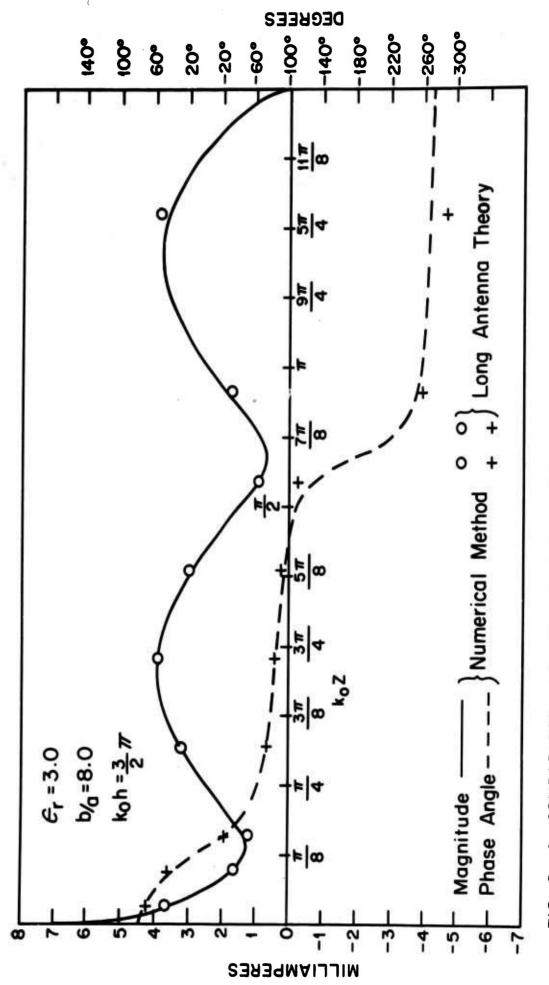


A comparison of the current distribution obtained by (18) with that obtained by the numerical method can further confirm the theory. The longest structure for which calculations were made in [1] is $h=3/4 \, \lambda_0$. With b/a=8, which has the largest transmission current, smallest radiation current among the three cases, the current distributions are compared in Fig. 6. Certainly $h=3/4 \, \lambda_0$ is not very long, however the agreement is not bad, except near the end where the theory yields no answer.

V CONCLUSIONS

With the use of the reflection coefficient for the transmission current an approximate solution for the long dielectric-coated antenna has been found. In general, the longer the antenna, the thicker the coating, and the higher the dielectric constant, the more accurate will the results be. The minimum length required before the theory can be applied depends on the required accuracy. As a rough estimate a finite length h is acceptable if the radiation current at a distance h along an infinitely long antenna is smaller than 5 per-cent of the transmission current at that point.

On the other hand as the coating becomes thinner and thinner, the relative magnitude of the transmission current decreases and the minimum length required before the theory can be applied becomes greater and greater. In the limit, as the coating goes to zero, the transmission current vanishes and the theory ceases to exist.



A COMPARISON OF CURRENT DISTRIBUTION BY LONG ANTENNA THEORY TO NUMERICAL METHOD FIG. 6

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Security Classification DOCUMENT CONTROL DATA - R & D Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified) ORIGINATING ACTIVITY (Corporate author) 28. REPORT SECURITY CLASSIFICATION Unclassified Cruft Laboratory 2b. GROUP Division of Engineering and Applied Physics Harvard University, Cambridge, Massachusetts 3 REPORT TITLE A THEORETICAL STUDY OF A LONG DIELECTRIC-COATED CYLINDRICAL ANTENNA 4 DESCRIPTIVE NOTES (Type of report and, inclusive dates) Interim technical report 5 AUTHOR(S) (First name, middle initial, last name) Chung-Yu Ting 6. REPORT DATE 78, TOTAL NO. OF FAGES 7b. NO. OF REFS 5 January 1967 94. ORIGINATOR'S REPORT NUMBER(5) SA. CONTRACT OR GRANT NO. Nonr - 1866(32)Technical Report No. 517 b. PROJECT NO. NR - 371 - 016 9b. OTHER REPORT NO(5) (Any other numbers that may be assigned this report) d. 10. DISTRIBUTION STATEMENT Reproduction in whole or in part is permitted by the U. S. Government. Distribution of this document is unlimited. 11. SUPPLEMENTARY NOTES 12. SPONSORING MILITARY ACTIVITY Office of Naval Research 13. ABSTRACT By applying the Wiener-Hopf technique, the reflection coefficient of the transmission current at the end of a dielectric-coated antenna can be expressed in a single integral form. This result, when used with the solution of an infinite dielectric-coated cylindrical antenna, yields the input admittance and the current distribution of a long dielectric coated antenna. It is found that unlike the locus of the admittance of a bare cylindrical antenna which converges to a point as the antenna gets longer and longer, the locus of the input admittance of the dielectriccoated antenna becomes a circle. Also, due to the reflection of the transmission current back and forth, a standing wave with surface wave length is formed along the antenna.

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